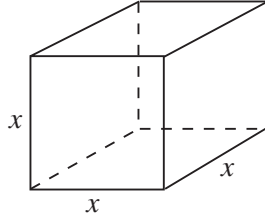


9.4 Surface Area and Volume of 3-D Shapes

In this section we calculate the volume and surface area of 3-D shapes such as *cubes*, *cuboids*, *prisms* and *cylinders*.

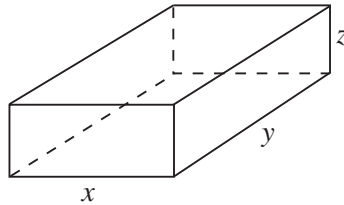
Cube



$$\text{Volume} = x^3$$

$$\text{Surface area} = 6x^2$$

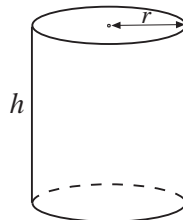
Cuboid



$$\text{Volume} = xyz$$

$$\text{Surface area} = 2xy + 2xz + 2yz$$

Cylinder



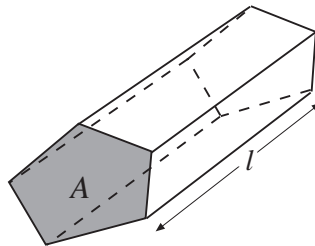
$$\text{Volume} = \pi r^2 h$$

$$\text{Area of curved surface} = 2\pi r h$$

$$\text{Area of each end} = \pi r^2$$

$$\text{Total surface area} = 2\pi r h + 2\pi r^2$$

Prism



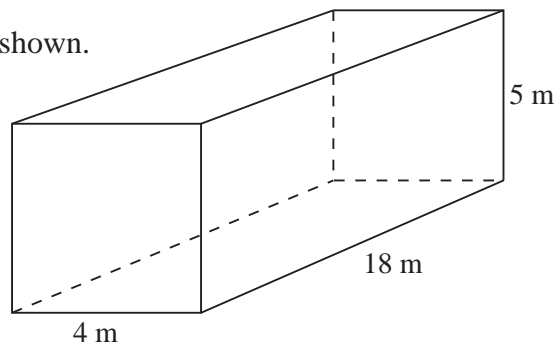
A prism has a uniform cross-section

$$\begin{aligned} \text{Volume} &= \text{area of cross-section} \times \text{length} \\ &= Al \end{aligned}$$



Example 1

- (a) Calculate the *volume* of the cuboid shown.
 (b) Calculate the *surface area* of the cuboid shown.



Solution

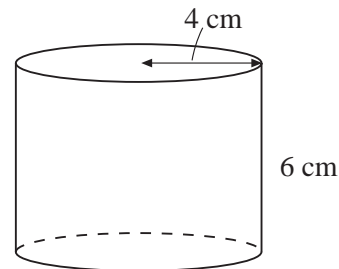
(a) Volume = $4 \times 18 \times 5$
 $= 360 \text{ m}^3$

(b) Surface area = $(2 \times 4 \times 18) + (2 \times 4 \times 5) + (2 \times 5 \times 18)$
 $= 144 + 40 + 180$
 $= 364 \text{ m}^2$



Example 2

Calculate the *volume* and total *surface area* of the cylinder shown.



Solution

Volume = $\pi r^2 h = \pi \times 4^2 \times 6 = 96\pi$
 $= 301.5928947 \text{ cm}^3$
 $= 302 \text{ cm}^3$ (to 3 significant figures)

Area of curved surface = $2\pi r h = 2 \times \pi \times 4 \times 6$
 $= 48\pi$
 $= 150.7964474 \text{ cm}^2$

Area of each end = $\pi r^2 = \pi \times 4^2$
 $= 16\pi$
 $= 50.26548246 \text{ cm}^2$

Total surface area = $150.7964474 + (2 \times 50.26548246)$
 $= 251.3274123 \text{ cm}^2$
 $= 251 \text{ cm}^2$ (to 3 significant figures)

Note: From the working we can see that the area of the curved surface is 48π , and that the area of each end is 16π . The total surface area is therefore

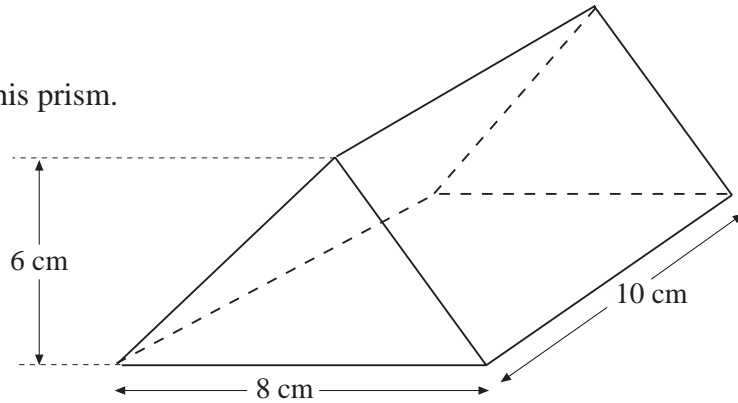
$$48\pi + (2 \times 16\pi) = 80\pi = 251.3274123 \text{ cm}^2$$

$$= 251 \text{ cm}^2 \text{ (to 3 significant figures)}$$



Example 3

Calculate the *volume* of this prism.



Solution

$$\text{Area of end of prism} = \frac{1}{2} \times 8 \times 6$$

$$= 24 \text{ cm}^2$$

$$\text{Volume of prism} = 24 \times 10$$

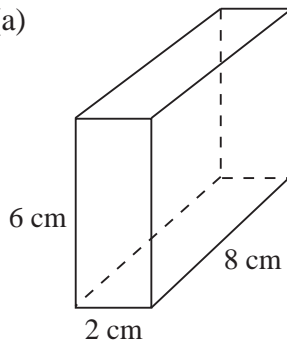
$$= 240 \text{ cm}^3$$



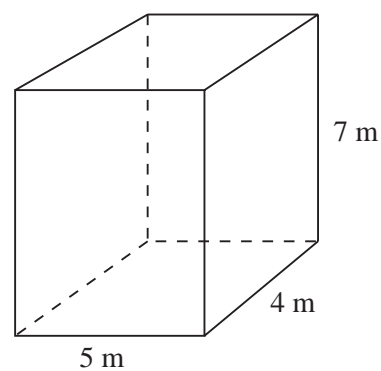
Exercises

1. Calculate the *volume* and *surface area* of each of the following cuboids:

(a)

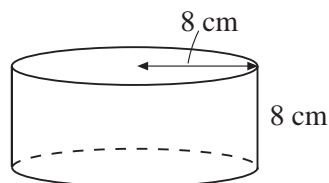


(b)

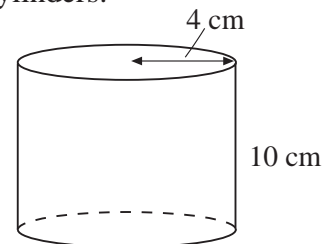


2. Giving your answers correct to 3 significant figures, calculate the *volume* and *total surface area* of each of the following cylinders:

(a)

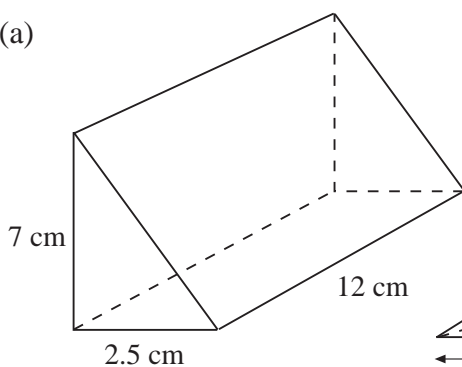


(b)

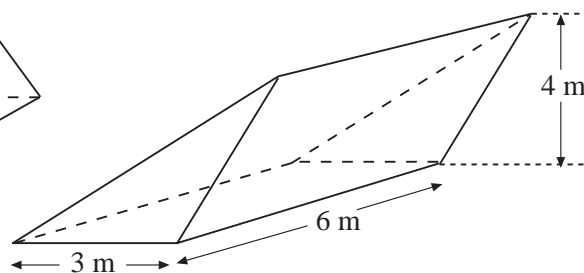


3. Calculate the *volume* of each of the following prisms:

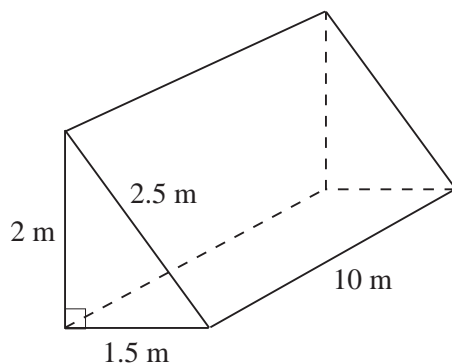
(a)



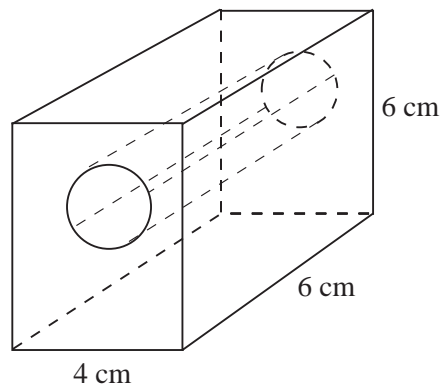
(b)



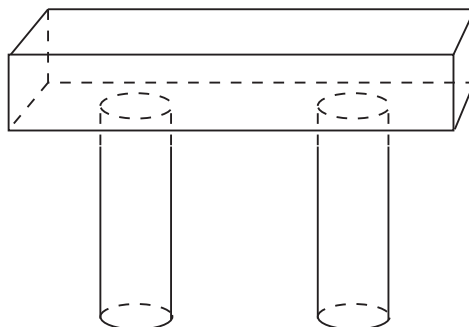
4. Calculate the *volume* and *surface area* of the following prism:



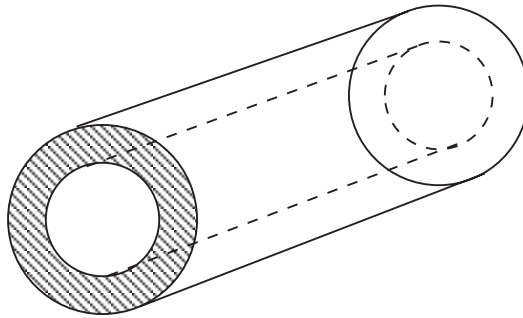
5. The diagram shows a wooden block that has had a hole drilled in it. The diameter of the hole is 2 cm.
Calculate the *volume* of this solid, giving your answer correct to 2 decimal places.



6. A concrete beam is to rest on two concrete pillars. The beam is a cuboid with sides of length 0.5 m, 3 m and 0.4 m.
The pillars have diameter 0.4 m and height 2 m.
Calculate the *total volume* of concrete needed to make the beam and the pillars. Round your answer to a sensible level of accuracy.

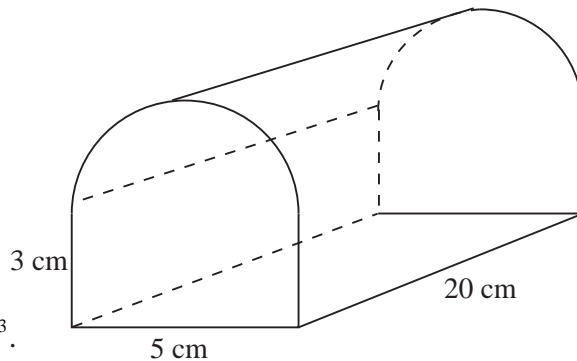


7. The diagram shows the cross-section of a pipe of length 50 cm. The inner diameter of the pipe is 20 cm and the outer diameter is 30 cm.



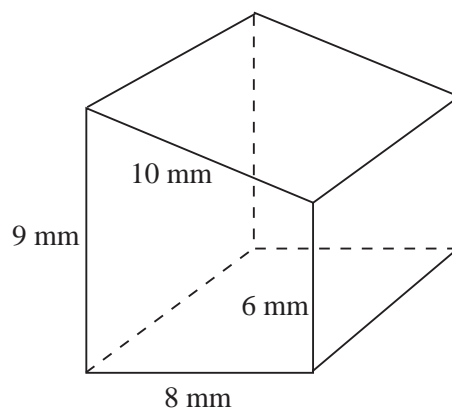
- (a) Calculate the *volume* of metal needed to make the pipe. Round your answer to a sensible level of accuracy.
- (b) Calculate the *total surface area* of the pipe, including the inside surface. Round your answer to a sensible level of accuracy.

8. The diagram shows a prism. The cross-section of the prism consists of a rectangle and a semicircle.



- (a) Calculate the *volume* of the prism. Give your answer to the nearest cm^3 .
- (b) Calculate the *total surface area* of the prism. Give your answer to the nearest cm^2 .

9. The volume of the prism shown is 720 mm^3 .

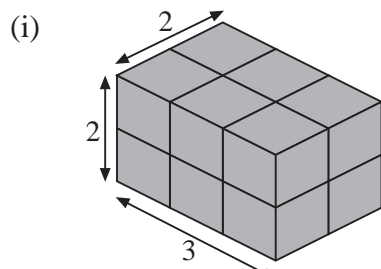


- (a) Determine the *length* of the prism.
- (b) Calculate the *surface area* of the prism.

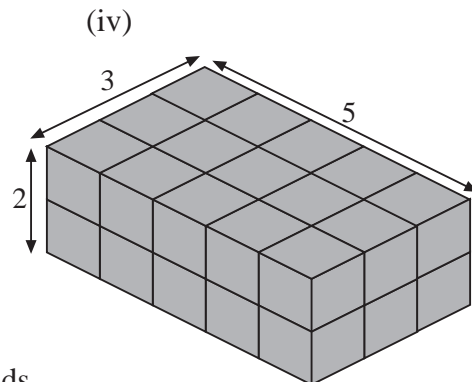
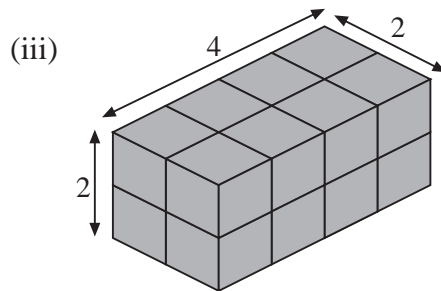
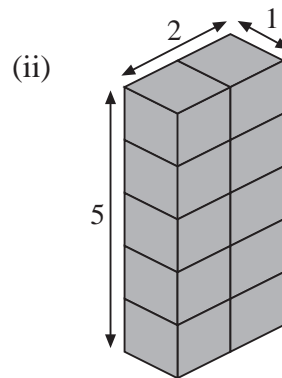
10. A cylinder has a diameter of 12 cm and a curved surface area of 132π or 415 cm^2 (to 3 significant figures).
- Determine the *height* of the cylinder.
 - Calculate the *volume* of the cylinder, giving your answer to the nearest cm^3 .

11. (a) These cuboids are made from small cubes. Write *how many small cubes* there are in each cuboid.

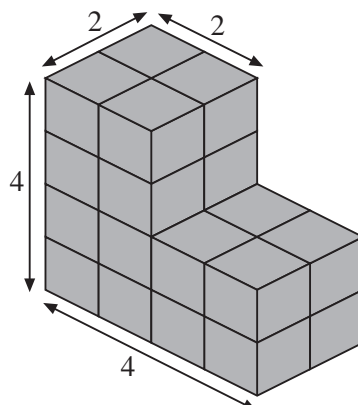
The first is done for you.



Cube (i) is made from 12 small cubes.

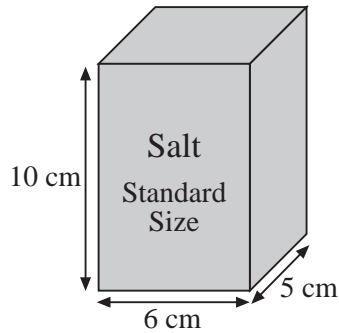


- (b) This shape is made with two cuboids. Write *how many cubes* there are in this shape.

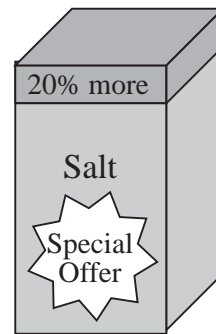




12. (a) What is the volume of this *standard size* box of salt?



- (b) What is the volume of this *special offer* box of salt, which is 20% bigger?



The standard size box contains enough salt to fill up 10 salt pots.

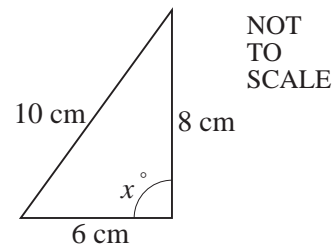


- (c) How many salt pots may be filled up from the *special offer* box of salt?

(KS3/96/Ma/Tier 5-7/P2)

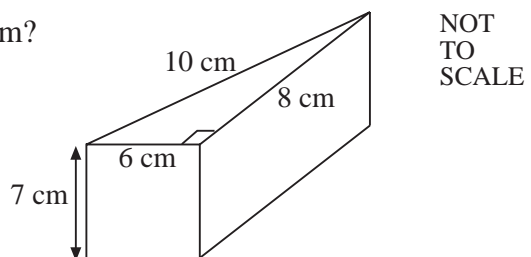
13. (a) Look at this triangle.

Show working to explain why angle x must be a right angle.

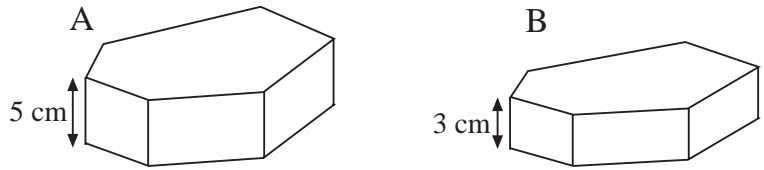


- (b) What is the volume of this prism?

You must show each step in your working.



- (c) Prisms A and B have the same cross-sectional area.



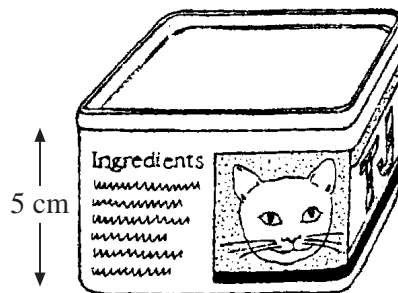
NOT TO SCALE

Copy and complete the table:

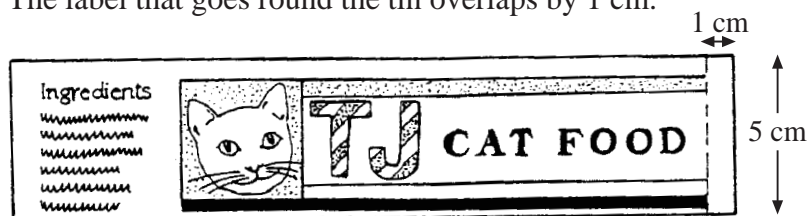
	<i>Prism A</i>	<i>Prism B</i>
height	5 cm	3 cm
volume	200 cm ³ cm ³

(KS3/99/Ma/Tier 5-7/P1)

14. TJ's Cat Food is sold in tins shaped like this. Each tin has an internal height of 5 cm.



- (a) The area of the lid of the tin is 35 cm².
Work out the volume of cat food that the tin contains.
- (b) The label that goes round the tin overlaps by 1 cm.



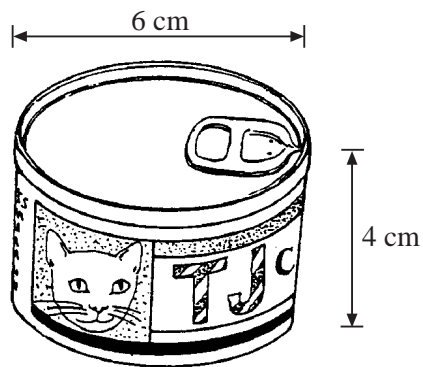
NOT TO SCALE

The area of the label is 134 cm².

Work out the distance around the tin.

Show your working.

TJ's Cat Food plans to use tins that are the shape of cylinders.
The internal measurements of a tin are shown.



- (c) Work out the volume of cat food that the tin contains.
Show your working.

(KS3/95/Ma/Levels 5-7/P2)